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1983 J. Phys. A: Math. Gen. 16 L279

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J. Phys. A: Math. Gen. **16** (1983) L279–L284. Printed in Great Britain

LETTER TO THE EDITOR

Electroweak effects in p-wave superconductors

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Received 7 February 1983

Abstract. The unification of electromagnetism and weak interactions leads to anisotropic mixing of the electroweak gauge bosons in p-wave superconductors. It also leads to parity violating components in the gap matrix.

In the standard electroweak theory (Glashow 1961, Weinberg 1967, Salam 1968) a locally $SU(2)\times U(1)$ gauge invariant theory is spontaneously broken down to the local electromagnetic gauge invariance of the vacuum. The pairing of electrons in a superconductor means that the (BCS) ground state breaks even this residual gauge invariance. The relevant energy scales of the two symmetry breakdowns differ enormously, with the electroweak breaking characterised by a vacuum expectation value of order 10^{11} eV, and the electromagnetic breaking by a gap of order 1 eV. For most purposes, therefore, the electroweak symmetry is completely irrelevant in superconductivity. However, when we study parity violation in superconductors the electroweak gauge invariance might lead to observable effects. This is because the (neutral) vector boson mass eigenstates in the superconductor $(\tilde{Z}, \tilde{\gamma})$ are coherent mixtures of the vacuum mass eigenstates (Z, γ) (Dereli *et al* 1982) and both $\tilde{Z}$ and $\tilde{\gamma}$ have parity violating interactions. In a previous paper (Bailin and Love 1982a) we studied this effect in ordinary superconductors, and suggested that the Z–γ mixing phenomenon generates possibly observable effects in experiments designed to detect parity violation using the Josephson effect (Vainshtein and Khriplovich 1975). More precisely, we showed that the Vainshtein–Khriplovich effect, which predicts a deviation from the flux quantisation condition, is itself modified by the mixing of the gauge bosons.

The experimental tests are difficult, and may require the use of Chevrel superconductors which remain superconducting at very high fields (Fischer 1975). However, the phenomenon is of great intrinsic interest, as a macroscopic manifestation of the gauge fields of weak and electromagnetic interactions. In this letter, we extend the discussion of Z–γ mixing to p-wave superconductors where the gauge bosons may mix in a spatially anisotropic way.

However, we discuss first an effect not considered by Dereli *et al* (1982) or Bailin and Love (1982a), namely the presence of parity violating covariants in the gap matrix, induced by the weak interactions. This discussion is presented only for pairing with total angular momentum $J=0$ and is given both for the case of an s-wave superconductor $(J^P=0^+)$ and for the case of a p-wave superconductor $(J^P=0^-)$, where P denotes parity. The discussion of Z–γ mixing effects follows this and is presented for a general p-wave spin triplet pair, which need not have $J=0$.

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The most general form of the electron gap matrix which is consistent with Fermi statistics and which has $J = 0$ is given by

$$\Delta(\boldsymbol{k}, \boldsymbol{K}) = \Delta_1\gamma_5 + \Delta_2\boldsymbol{n} \cdot \boldsymbol{\gamma}\gamma_0\gamma_5 + \Delta_3\gamma_0\gamma_5 + \Delta_4 I + \Delta_5\boldsymbol{n} \cdot \boldsymbol{\gamma}\gamma_0 + \Delta_6\boldsymbol{n} \cdot \boldsymbol{\gamma}, \tag{1}$$

where $\boldsymbol{n}$ is a unit vector in the direction of the relative momentum $\boldsymbol{k}$ of the electrons in the pair, and $\boldsymbol{K}$ is their total centre-of-mass momentum. The coefficients Δ_i ($i = 1 \ldots 6$) are in general functions of $\boldsymbol{K}$ (but not of $\boldsymbol{n}$). In the non-relativistic (NR) limit this reduces to

$$\Delta = \Delta_1 - \Delta_3 - (\Delta_5 + \Delta_6)\boldsymbol{n} \cdot \boldsymbol{\sigma}, \tag{2}$$

showing that the terms proportional to Δ_1, Δ_3 characterise s-wave pairing ($P = +$), while those proportional to Δ_5, Δ_6 describe p-wave pairs ($P = -$). In the present context, therefore, we are concerned with a system in which

$$\Delta_5, \Delta_6 \gg \Delta_1, \Delta_3 \tag{3}$$

and in which Δ_1, Δ_3 are only non-zero because the pairing interaction is not parity invariant, since weak interactions violate parity. The general technique for deriving and solving the gap equation has been thoroughly explained in other publications (Bailin and Love 1982b, c) and we merely quote the results. In the notation of these earlier papers, the gap equation has the form

$$\Delta(\boldsymbol{k}, \boldsymbol{K}) = \tfrac{1}{2}g^2 \int d^3q(2\pi)^{-3}D_{AB}(\boldsymbol{k} - \boldsymbol{q})\tilde{\Gamma}^A R\Gamma^B, \tag{4}$$

where $-ig\Gamma^A$ describes the electron–phonon interaction, and D_{AB} is the exchanged phonon's propagator. Assuming scalar phonon exchange, we take

$$g\Gamma^A = g_\varphi I \tag{5a}$$

and the propagator to have the form

$$D_{AB}(\boldsymbol{k} - \boldsymbol{q}) = \tilde{V}(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{q}}) \tag{5b}$$

with

$$|\boldsymbol{k}| = |\boldsymbol{q}| = p_F \tag{5c}$$

where p_F is the Fermi momentum. In addition we include a parity violating interaction which will be generated by Z exchange between the electrons of the pair. Thus the weak interaction is modelled by an additional contribution with

$$g\Gamma^A = \gamma_\alpha(g_V + g_A\gamma_5) \tag{6a}$$

where

$$g_V = (g/2\cos\theta_w)(2\sin^2\theta_w - \tfrac{1}{2}), \qquad g_A = (g/2\cos\theta_w)(\tfrac{1}{2}) \tag{6b, c}$$

with θ_w the weak mixing angle and $g = e/\sin\theta_w$ the semi-weak coupling constant in the standard model. The (known) propagator for Z exchange is of the form

$$D_{AB}(\boldsymbol{k} - \boldsymbol{q}) = g_{\alpha\beta}\tilde{X}(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{q}}) \tag{7}$$

where $\boldsymbol{k}$ and $\boldsymbol{q}$ satisfy (5c). Proceeding as in Bailin and Love (1982b, c), we find that the components $\Delta_1, \ldots, \Delta_6$ of the gap are all proportional to a single gap combination, which in the 0^--dominant case is given by

$$d \equiv \Delta_5 - \frac{p_F}{\mu}\Delta_4 + \frac{m}{\mu}\Delta_6 + 4\frac{g_V g_A p_F X_0}{g_\varphi^2 \mu (V_1 - V_0)}\left(\Delta_1 - \frac{m}{\mu}\Delta_3 - \frac{p_F}{\mu}\Delta_2\right), \tag{8}$$

where m is the electron mass and μ is the (relativistic) chemical potential given by

$$\mu = (p_F^2 + m^2)^{1/2}. \tag{9}$$

In (8) the quantities V_0, V_1 and X_0 are essentially partial wave amplitudes of the potentials given in (5*b*) and (7):

$$V_i \equiv \int \frac{d\Omega_q}{4\pi} P_i(\hat{k}\cdot\hat{q})\tilde{V}(\hat{k}\cdot\hat{q}), \qquad X_0 \equiv \int \frac{d\Omega_q}{4\pi}\tilde{X}(\hat{k}\cdot\hat{q}). \tag{10a, b}$$

Solving the gap equation in the Ginzburg–Landau region, close to the critical temperature T_c, gives

$$\Delta = [(\mu^2 + m^2)V_1 - p_F^2 V_0]^{-1}\Big(p_F V_0 I + \mu V_1 \boldsymbol{n}\cdot\boldsymbol{\gamma}\gamma_0 + m V_1 \boldsymbol{n}\cdot\boldsymbol{\gamma}$$
$$+ \frac{4 g_V g_A p_F X_0}{g_\varphi^2 \mu (V_1 - V_0)}[(2V_1 - V_0)\mu\gamma_5 + p_F V_1 \boldsymbol{n}\cdot\boldsymbol{\gamma}\gamma_0\gamma_5 - m V_0 \gamma_0\gamma_5]\Big)\mu d \tag{11}$$

with the overall scale d given by minimising

$$\mathscr{F} = \frac{1}{2}\frac{\mu p_F}{\pi^2}\left(t d^* d + \frac{7\zeta(3)}{16\pi^2 (k_B T_c)^2}(d^* d)^2 + \frac{7\zeta(3) p_F^2}{48\pi^2 \mu^2 (k_B T_c)^2}(\nabla d^*)\cdot(\nabla d)\right) \tag{12a}$$

where

$$t = (T - T_c)/T_c. \tag{12b}$$

Notice that in the NR limit $p_F \ll m \approx \mu$. It follows using (2) that

$$\Delta \to -d\boldsymbol{n}\cdot\boldsymbol{\sigma} \tag{13}$$

so that the parity mixing occurs only as a relativistic effect of order v/c.

We note that the same calculation is readily adapted to handle the case of ordinary s-wave superconductors in which the gap is predominantly $J^P = 0^+$. Then all components $\Delta_1, \ldots, \Delta_6$ are proportional to

$$e \equiv \Delta_1 - \frac{m}{\mu}\Delta_3 - \frac{p_F}{\mu}\Delta_2 - \frac{4 g_V g_A p_F X_0}{g_\varphi^2 \mu (V_1 - V_0)}\left(\Delta_5 - \frac{p_F}{\mu}\Delta_4 + \frac{m}{\mu}\Delta_6\right) \tag{14}$$

and the solution is

$$\Delta = [(\mu^2 + m^2)V_0 - p_F^2 V_1]^{-1}\{\mu V_0 \gamma_5 + p_F V_1 \boldsymbol{n}\cdot\boldsymbol{\gamma}\gamma_0\gamma_5 - m V_0 \gamma_0\gamma_5$$
$$- [4 g_V g_A X_0 / g_\varphi^2 \mu (V_1 - V_0)]$$
$$\times [2\mu^2 (V_1 - V_0) + p_F^2 V_0 + \mu p_F V_1 \boldsymbol{n}\cdot\boldsymbol{\gamma}\gamma_0 + m p_F V_1 \boldsymbol{n}\cdot\boldsymbol{\gamma}]\}\mu e, \tag{15}$$

where e is again given by minimising the free energy (12) with d replaced by e. As before, the parity violation in the gap is of order v/c and disappears in the static limit:

$$\Delta \to e. \tag{16}$$

The parity violating admixture in the gap matrix, together with the parity violating Hamiltonian for the weak force between electrons, will lead, in the p-wave case, to a reduction in the degeneracy of the order parameter, as a doubly weak effect. We hope it may be possible to devise more sensitive tests of a parity violating admixture in the gap. In any case, there is a singly weak effect in the mixing of the neutral electroweak gauge bosons which occurs when the derivative $\partial_\alpha d$ in (12) is replaced by the $SU(2)\times U(1)$ covariant derivative $D_\alpha d$. To determine this we must find the gauge transformation properties of d, and these are determined by the transformation properties of the various bilinear covariants appearing in (1). As before (Bailin and Love 1982a), we consider only those transformations which transform an electron state into an electron state, since there is no electron–neutrino condensate. Then the gauge covariant derivative of the bilinear $\bar{e}^c\Gamma e$ is (for an arbitrary γ-matrix Γ):

$$D_\alpha(\bar{e}^c\Gamma e) = (\partial_\alpha - iP_\alpha)(\bar{e}^c\Gamma e) + \tfrac{1}{2}iQ_\alpha\bar{e}^c\{\Gamma, \gamma_5\}e, \tag{17a}$$

where P_α and Q_α are combinations of the neutral gauge boson fields:

$$P_\alpha = \tfrac{1}{2}(gW^3_\alpha + 3g'B_\alpha) = e \operatorname{cosec} 2\theta_w[(1-4\sin^2\theta_w)Z_\alpha + 2\sin 2\theta_w A_\alpha], \tag{17b}$$

$$Q_\alpha = \tfrac{1}{2}(gW^3_\alpha - g'B_\alpha) = e \operatorname{cosec} 2\theta_w Z_\alpha. \tag{17c}$$

Z_α, A_α are the neutral vector boson mass eigenstates in vacuo (but not, as we shall see, in a superconductor). It follows that

$$D_\alpha\Delta_1 = (\partial_\alpha + iP_\alpha)\Delta_1 - iQ_\alpha\Delta_4, \qquad D_\alpha\Delta_2 = (\partial_\alpha + iP_\alpha)\Delta_2 - iQ_\alpha\Delta_5, \tag{18a, b}$$

$$D_\alpha\Delta_3 = (\partial_\alpha + iP_\alpha)\Delta_3, \qquad D_\alpha\Delta_4 = (\partial_\alpha + iP_\alpha)\Delta_4 - iQ_\alpha\Delta_1, \tag{18c, d}$$

$$D_\alpha\Delta_5 = (\partial_\alpha + iP_\alpha)\Delta_5 - iQ_\alpha\Delta_2, \qquad D_\alpha\Delta_6 = (\partial_\alpha + iP_\alpha)\Delta_6. \tag{18e, f}$$

Then using (8) we find

$$D_\alpha d = \left(\partial_\alpha + iP_\alpha + \frac{8g_Vg_Ap_F^2X_0}{g_\varphi^2[(\mu^2+m^2)V_1 - p_F^2V_0]}iQ_\alpha\right)d \tag{19}$$

which gives

$$D_\alpha d \to (\partial_\alpha + iP_\alpha)d \tag{20}$$

in the NR limit. Thus in the $J^P = 0^-$ dominant p-wave pairing so far considered the Z–γ mixing in the NR limit is precisely that found in Bailin and Love (1982a).

However, this form of p-wave pairing is not the most general, nor even the most likely, to arise, as our experience of superfluid ^{3}He has shown (Leggett 1975). In general, for a p-wave spin triplet pair, independently of the total angular momentum J, we may write

$$\Delta = d_{\mu i}\sigma^\mu n^i \tag{21}$$

in the notation of (2). Evidently the 0^- pairing we have considered hitherto is described by

$$d_{\mu i} = d\delta_{\mu i}. \tag{22}$$

Our experience of ^{3}He suggests that we should also consider the following special cases.

(i) B-phase

$$d_{\mu i} = d\, e^{i\chi}R_{\mu i} \tag{23}$$

where d and χ are real and $R_{\mu i}$ is a rotation matrix. (This includes (22) as a special case.)

(ii) Planar phase

$$d_{\mu i} = d\ \mathrm{e}^{\mathrm{i}\chi}\varepsilon_{\mu i k}\omega^{k} \tag{24}$$

where d and χ are real and $\boldsymbol{\omega}$ is a real unit vector.

(iii) A phase

$$d_{\mu i} = d\beta_{\mu}(1/\sqrt{2})(\alpha_{1i} + \mathrm{i}\alpha_{2i}) \tag{25}$$

where d is real, $\boldsymbol{\beta}$ is a real unit vector, and $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$ are real orthonormal vectors.

(iv) A_1 phase

$$d_{\mu i} = d\tfrac{1}{2}(\beta_{1\mu} + \mathrm{i}\beta_{2\mu})(\alpha_{1i} + \mathrm{i}\alpha_{2i}) \tag{26}$$

where d is real, $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$ are real orthonormal vectors, and so are $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$.

In these more general cases the derivative terms in the free energy have a more complicated form than that given in (12), which describes only the $J^P = 0^-$ case. The gradient free energy has the general structure

$$\mathscr{F}_{\text{grad}} = \sum_{\mu=1}^{3} (\tfrac{1}{2}K_{\mathrm{L}}|\boldsymbol{\nabla}\cdot\boldsymbol{d}_{\mu}|^2 + \tfrac{1}{2}K_{\mathrm{T}}|\boldsymbol{\nabla}\wedge\boldsymbol{d}_{\mu}|^2), \tag{27}$$

where $\boldsymbol{d}_{\mu}$ is the (complex) vector with components $d_{\mu i}$, and in weak coupling approximation

$$K_{\mathrm{T}} = \tfrac{1}{3}K_{\mathrm{L}} = 7\zeta(3)p_{\mathrm{F}}^3/80\pi^4(k_{\mathrm{B}}T_{\mathrm{c}})^2 m \tag{28}$$

(i.e. K_{T} is $\frac{6}{5}$ times the coefficient of $|\boldsymbol{\nabla}d|^2$ in (12)). When we replace the derivative $\boldsymbol{\nabla}$ by the covariant derivative $\boldsymbol{D}$ given in (20), we generate gauge boson mass terms additional to those generated by the Higgs doublet in the standard model. In fact the total neutral gauge boson mass Lagrangian is given by

$$-\mathscr{L}_{\mathrm{M}} = \tfrac{1}{2}m_Z^2 Z^i Z^i + \tfrac{1}{2}K_{\mathrm{T}} d_{\mu j}^* d_{\mu j} P^i P^i + \tfrac{1}{2}(K_{\mathrm{L}} - K_{\mathrm{T}}) d_{\mu i}^* d_{\mu j} P^i P^j, \tag{29a}$$

where

$$m_Z^2 = e^2 \operatorname{cosec}^2 2\theta_{\mathrm{w}}/\sqrt{2}G_{\mathrm{F}} \tag{29b}$$

gives the Z-boson mass in vacuo. The field combination P^i is given in (17b). The anisotropy, if it occurs, enters the mass matrix via the last term of (29a).

Since the additional mass terms are small compared with m_Z^2, we may work to first order in them. Then the mass eigenstates will in general have the form

$$\left.\begin{aligned} \tilde{Z}_{\alpha} &= Z_{\alpha} + x_{\alpha}A_{\alpha} \quad &(30a)\\ \tilde{A}_{\alpha} &= A_{\alpha} - x_{\alpha}Z_{\alpha} \quad &(30b)\end{aligned}\right\} \text{ no summation}$$

where Z and A are the vacuum eigenstates and the (small) mixing parameter x_{α} is expected to be anisotropic. The mass eigenvalues all have the form

$$m^2(\tilde{Z}_{\alpha}) = m_Z^2[1 + K_{\alpha}\sqrt{2}G_{\mathrm{F}}d^2(1 - 4\sin^2\theta_{\mathrm{w}})^2], \tag{31a}$$

$$m^2(\tilde{A}_{\alpha}) = 4e^2 K_{\alpha} d^2, \tag{31b}$$

with K_{α} in general anisotropic, and we find that in all cases

$$x_{\alpha} = K_{\alpha}\sqrt{2}G_{\mathrm{F}}d^2 2\sin 2\theta_{\mathrm{w}}(1 - 4\sin^2\theta_{\mathrm{w}}). \tag{32}$$

Of the four phases defined in (23)–(26) only the B phase has no anisotropy, since

$$R_{\mu i}R_{\mu j} = \delta_{ij}. \tag{33}$$

In this case the mixing and the mass eigenvalues are given by

$$K^{(B)} = K_L + 2K_T. \quad (34)$$

As the name implies, the planar phase has a planar symmetry (in the plane perpendicular to $\boldsymbol{\omega}$). In this plane the mass eigenstates, denoted $\tilde{Z}_\perp^{(P)}$, $\tilde{A}_\perp^{(P)}$ are given by

$$K_\perp^{(P)} = K_L + K_T. \quad (35)$$

However, in the direction parallel to $\boldsymbol{\omega}$ the masses and mixing are given by

$$K_\parallel^{(P)} = 2K_T. \quad (36)$$

As expected from (29*a*) the masses and the mixing are isotropic when $K_L = K_T$.

In the remaining two phases the anisotropy is quite independent of the vectors $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$ or $\boldsymbol{\beta}$. In both of these cases there is also a symmetry in the plane defined by $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ (perpendicular to $\boldsymbol{\alpha}_1 \wedge \boldsymbol{\alpha}_2$). In this plane

$$K_\perp^{(A)} = K_\perp^{(A_1)} = \tfrac{1}{2}(K_L + K_T), \quad (37)$$

while in the direction parallel to $\boldsymbol{\alpha}_1 \wedge \boldsymbol{\alpha}_2$ we find

$$K_\parallel^{(A)} = K_\parallel^{(A_1)} = K_T. \quad (38)$$

Again there is isotropy when $K_L = K_T$, as expected.

One way to look for these anisotropic effects would be to consider deviations from magnetic flux quantisation conditions. For example, in the case of the planar phase (24), if the superconducting loop is in the plane perpendicular to a uniform $\boldsymbol{\omega}$, then flux is quantised in the absence of the electroweak effect, studied here. Similarly, if we can prepare a loop with $\boldsymbol{\omega}$ parallel to $\boldsymbol{dl}$ everywhere, the flux is again quantised in the absence of electroweak effects.

We thank A J Leggett for suggesting that we should study p-wave superconductors, and G Barton and N Rivier for valuable comments. This research was supported in part by SERC under grant number NG 11283.

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